More Properties of Integrals- HW Problems

1. Evaluate the following definite integrals using symmetry (i.e. f odd/even) where applicable.

$$a. \quad \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x\cos(x) + \cos(x)) dx$$

$$b. \quad \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} x^2 \sin(x) \, dx$$

c.
$$\int_{-1}^{1} (2 - |x|) dx$$

Find the average value of the function on the given interval.

2.
$$f(x) = x^2 - 9$$
 on $[-3,3]$

3.
$$G(x) = \sin(x)$$
 on $[0, \pi]$

4.
$$h(x) = \cos(x)$$
 on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

5. The profit in thousands of dollars over time of a firm's new product is given by $P(t) = 4(-t^2 + 20t + 20)$, where t is in months. Find the average profit over the first 6 months.

Find all values of \boldsymbol{c} guaranteed by the mean value theorem for integrals.

6.
$$f(x) = \frac{1}{x^2}$$
 on [1,4].

7.
$$f(x) = -3x^2 + 12x + 2$$
 on [0,4].