The Mean Value Theorem- HW Problems

Determine if Rolle's theorem applies to $f(x)$ on the given interval. If it does, find all values c that satisfy the conclusion of Rolle's theorem. If it doesn't satisfy Rolle's theorem, explain why it doesn't.

1. $f(x) = x^2 - 4x$ on $[0,4]$

2. $f(x) = \sqrt{x} - \frac{1}{2}x$ on $[0,4]$

3. $f(x) = x^{\frac{2}{3}} - 3$ on $[-8.8]$

4. $f(x) = \cos(x)$ on $[0,2\pi]$

5. $f(x) = \tan(x)$ on $[0.\pi]$

Determine if the mean value theorem applies to $f(x)$ in the given interval. If it does, find all values c that satisfy the conclusion of the mean value theorem. If it doesn't satisfy the MVT, explain why it doesn't.

6. $f(x) = 2x^3 - 6x + 2$ on $[0,2]$

7. $f(x) = |x|$ on $[-2,3]$

8. $f(x) = x^{\frac{2}{3}}$ on $[0,8]$

9. $f(x) = 2\sin(x) + \sin(2x)$ on $[0,\pi]$

10. Show that the equation $x^3 - 27x + K = 0$ has at most one root in $[-2,2]$ (a root is a point c such that $c^3 - 27c + K = 0$). Hint: suppose $f(x) = x^3 - 27x + K$ has two roots in $[-2,2]$, c_1 and c_2. Now apply Rolle's theorem to $[c_1, c_2]$.