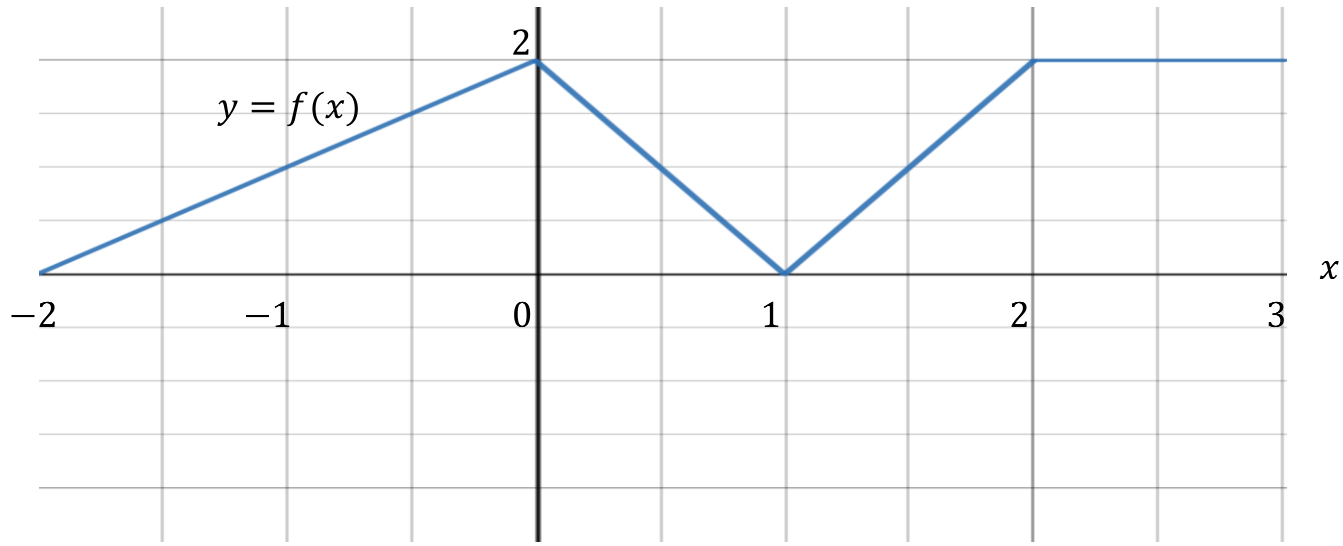
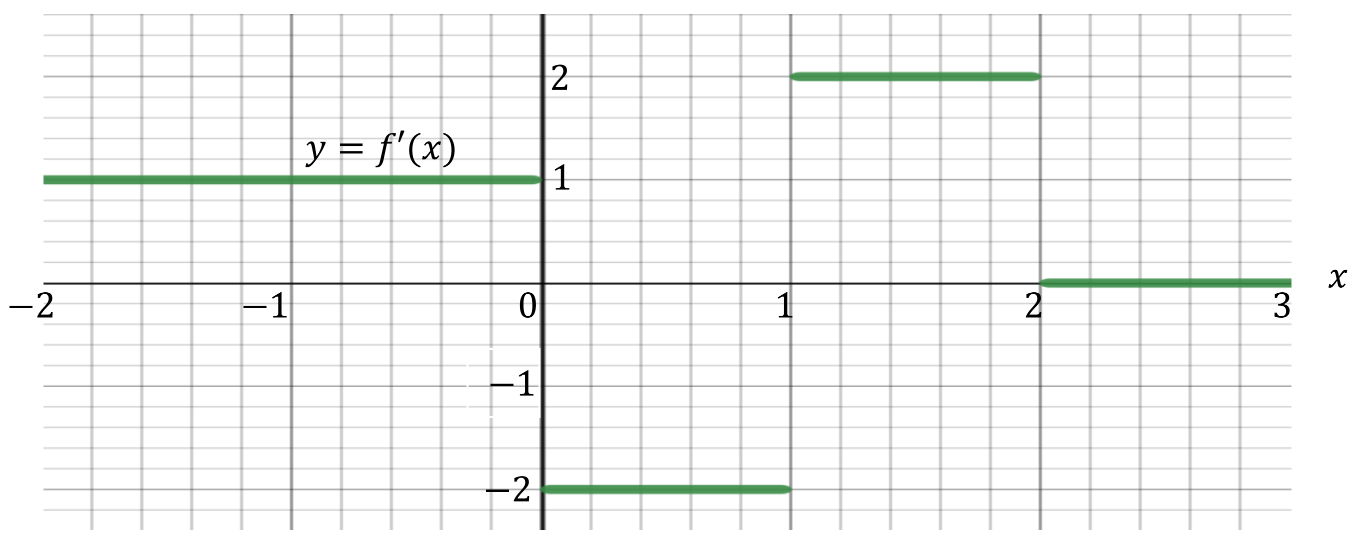
The Graph of $f'(x)$

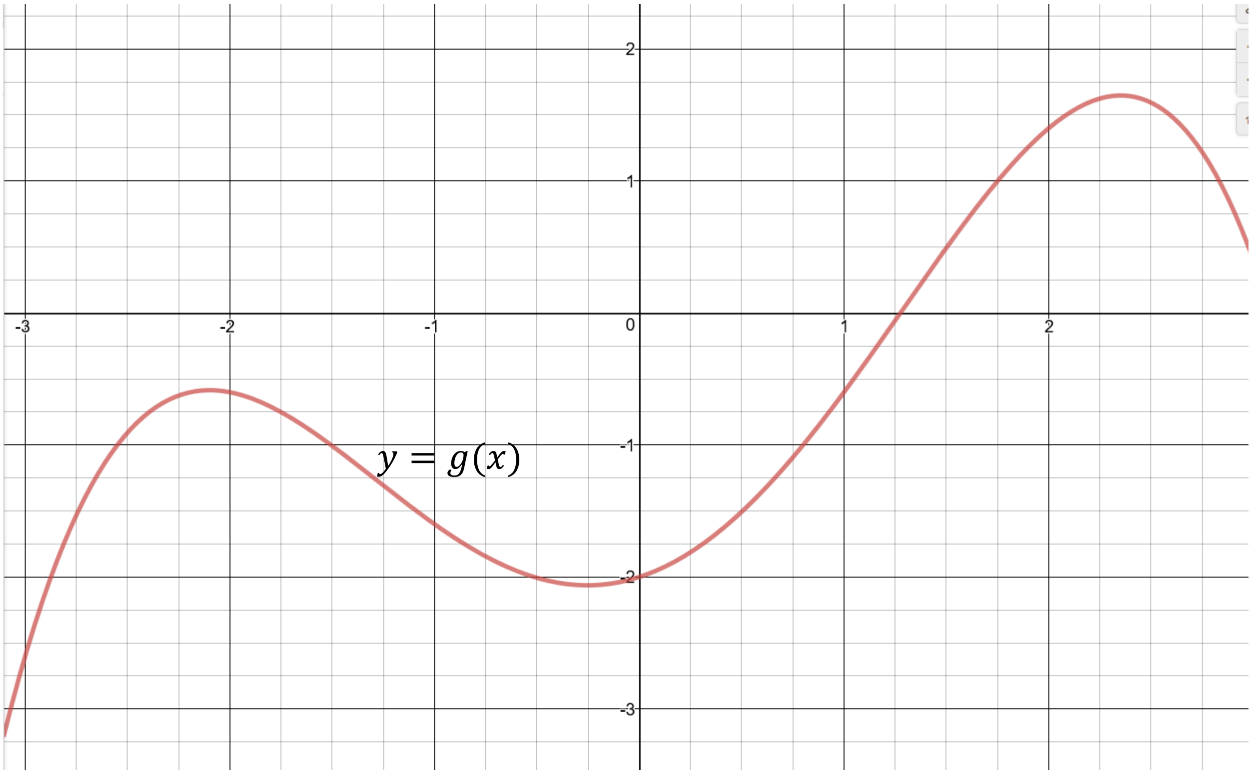
Notice that since the derivative at a point $x = a$ is the slope of the tangent line to the graph of $f(x)$ at $x = a$, we can say something about the graph of the derivative, $f'(x)$, simply by looking at the graph of $f(x)$.

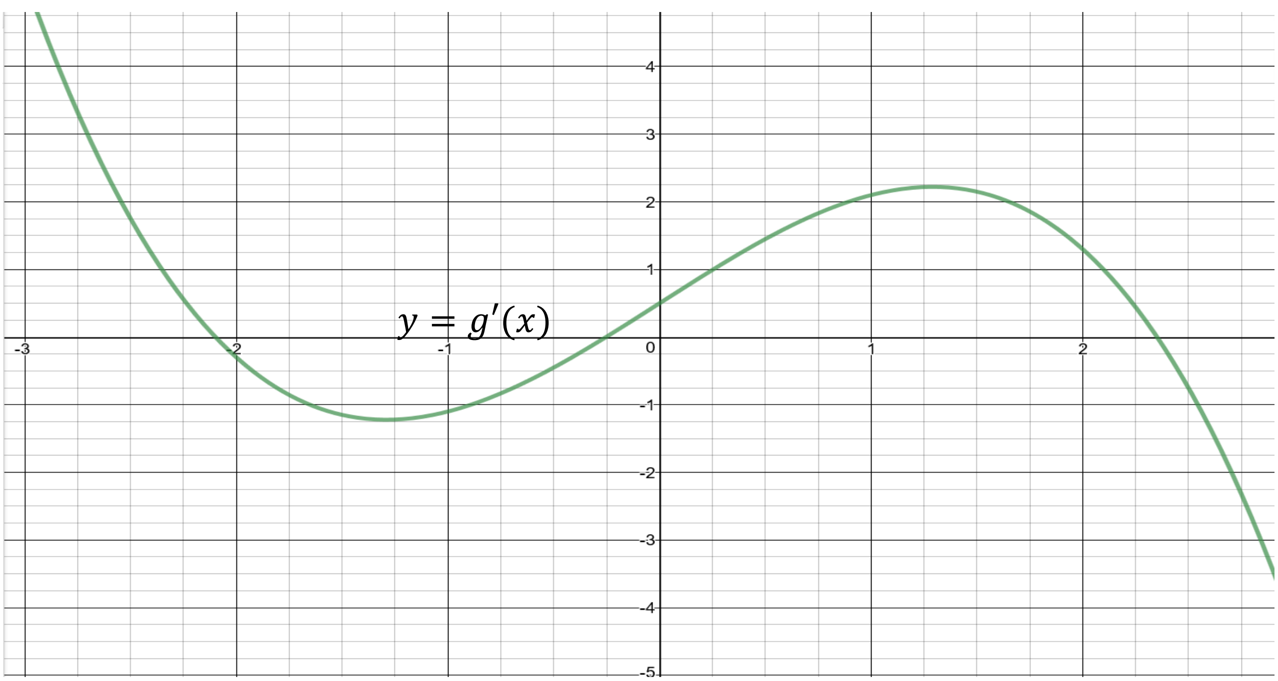
Ex. Below is the graph of $y = f(x)$. Sketch a graph of $f'(x)$.



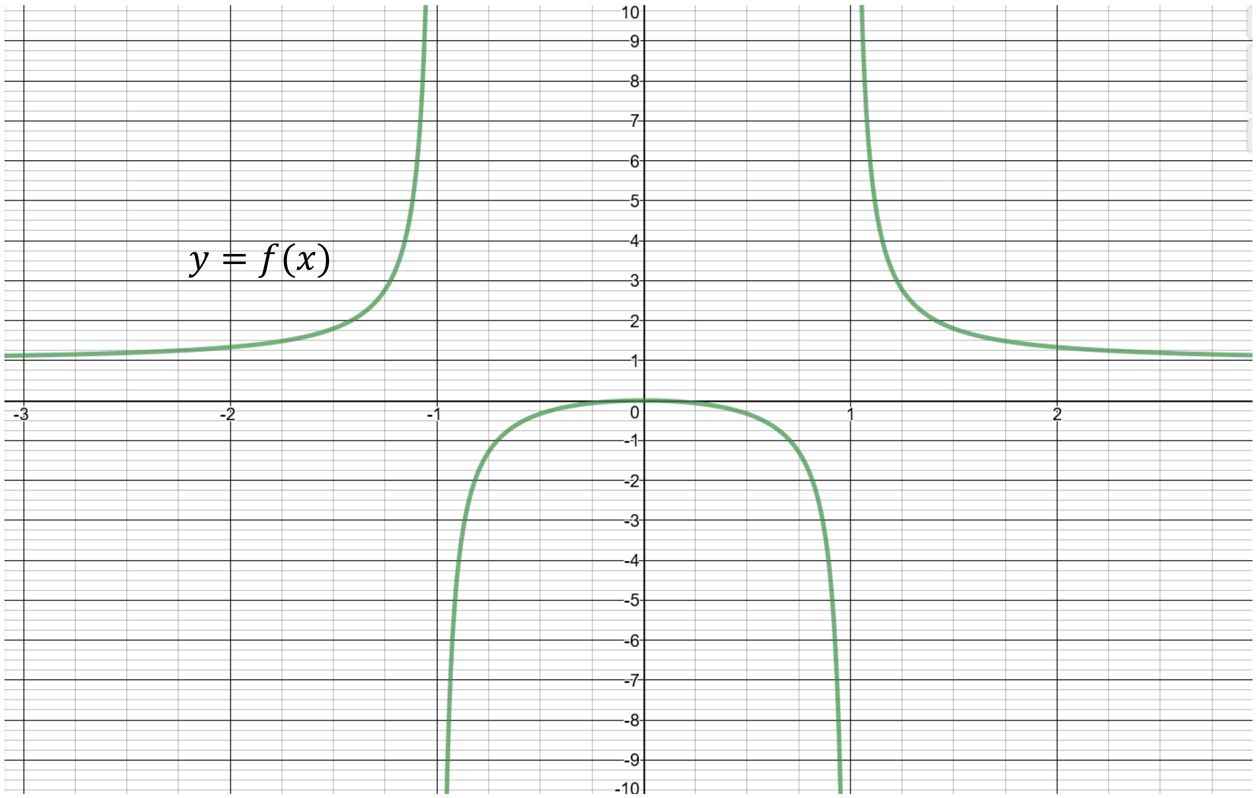


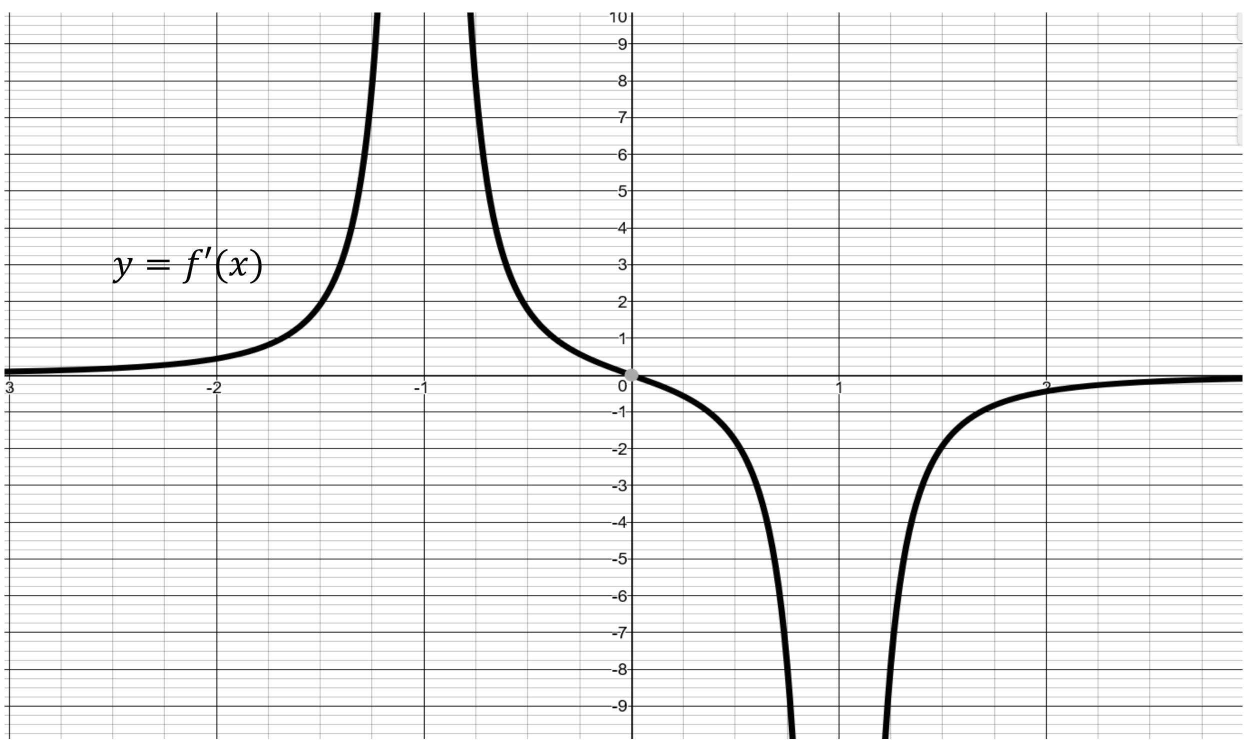
Ex. Below is the graph of $y = g(x)$. Sketch a rough graph of $g'(x)$.





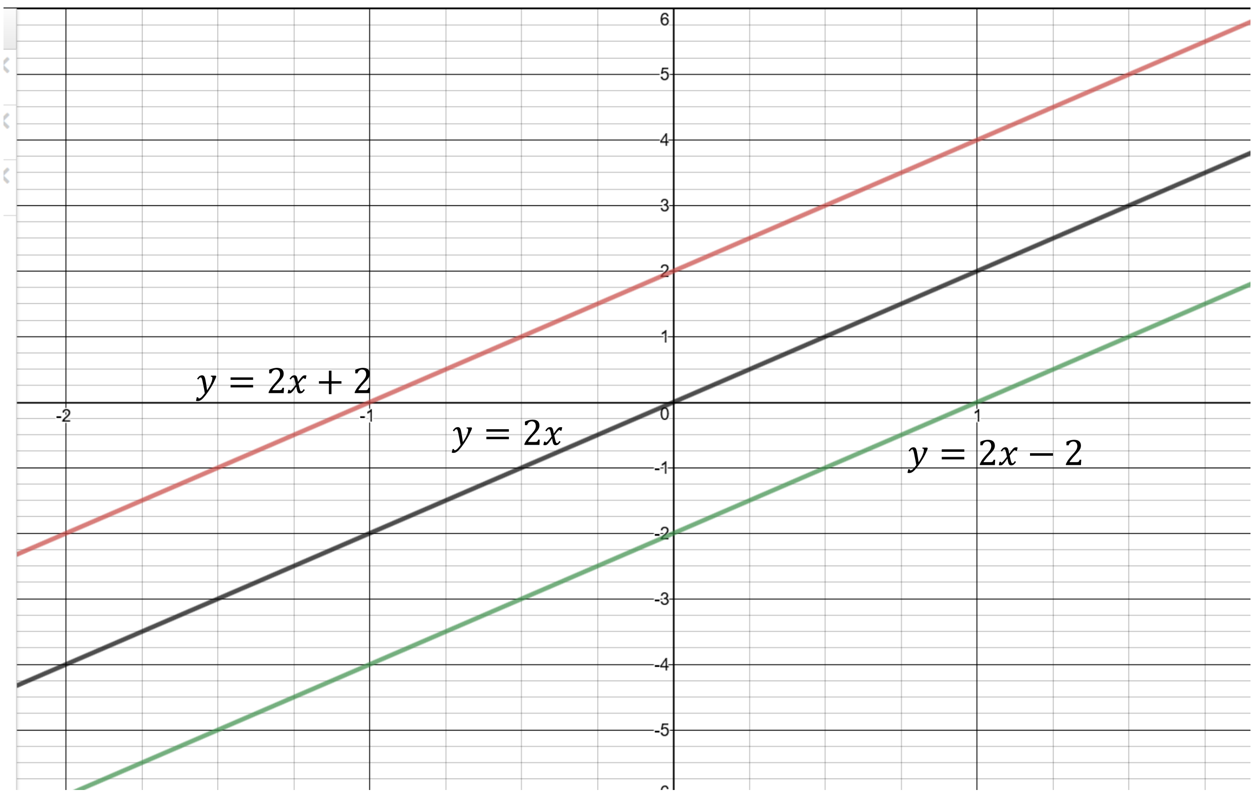
Ex. Below is the graph of $y = f(x)$. Sketch a rough graph of $f'(x)$.





Ex. Suppose $f'(x) = 2$ Sketch a possible graph of $f(x)$. Can there be more than one answer?

Yes!! Any line of the form $y = 2x + k$ works for any constant k.



Continuity Theorem:

Theorem: If f is differentiable at $x = a$, then f is continuous at $x = a$.

We want to show that if $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists then $\lim_{x\to a} f(x) = f(a)$, or equivalently, $\lim_{x\to a}(f(x) - f(a)) = 0$.

Notice that: $f(x) - f(a) = [\frac{f(x)-f(a)}{x-a}][(x-a)]$

So, $$\lim_{x\to a}(f(x) - f(a)) = \lim_{x\to a}([\frac{f(x)-f(a)}{x-a}][(x-a)])$$
$$= \lim_{x\to a}[\frac{f(x)-f(a)}{x-a}] \lim_{x\to a}(x-a)$$
$$= (f'(a))(0) = 0.$$

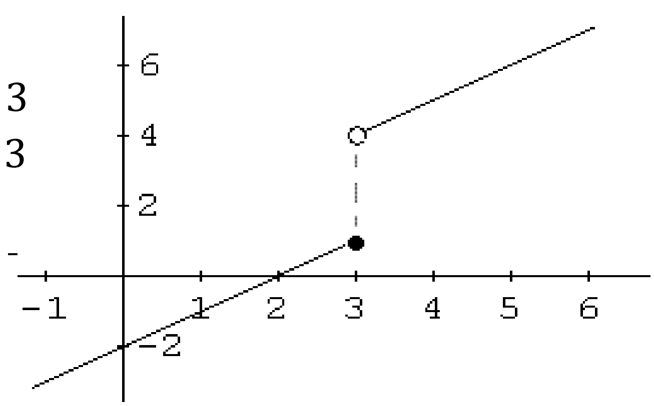
The converse of this theorem is not true. If a function is continuous at a point $x = a$, it doesn't have to be differentiable at that point (e.g. there could be a corner at $x = a$, like the function $y = |x|$ at $x = 0$).

However, if a function is not continuous at $x = a$, then it can't be differentiable at $x = a$.

Ways in which a function $f(x)$ can fail to be differentiable at $x = a$:

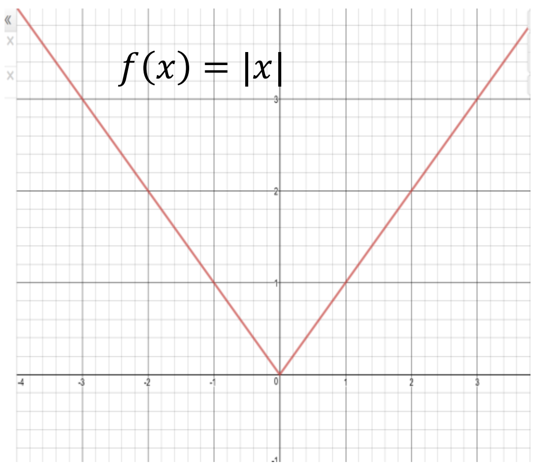
1. f is not continuous at $x = a$.

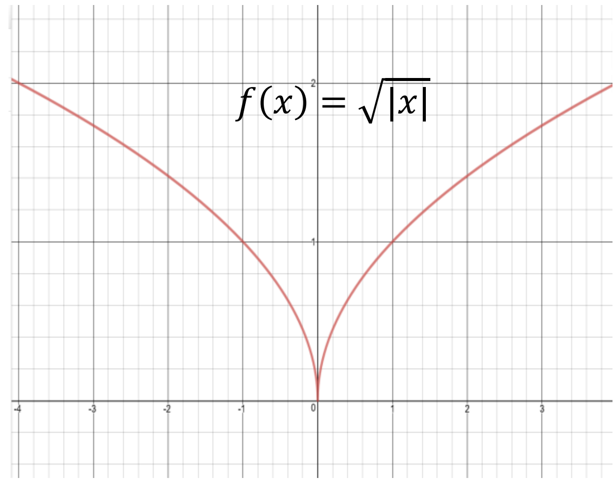
Ex. $f(x) = x + 1; \quad x > 3$
$\quad = x - 2; \quad x \le 3$
at $x = 3$.



2. f has a corner or cusp at $x = a$.

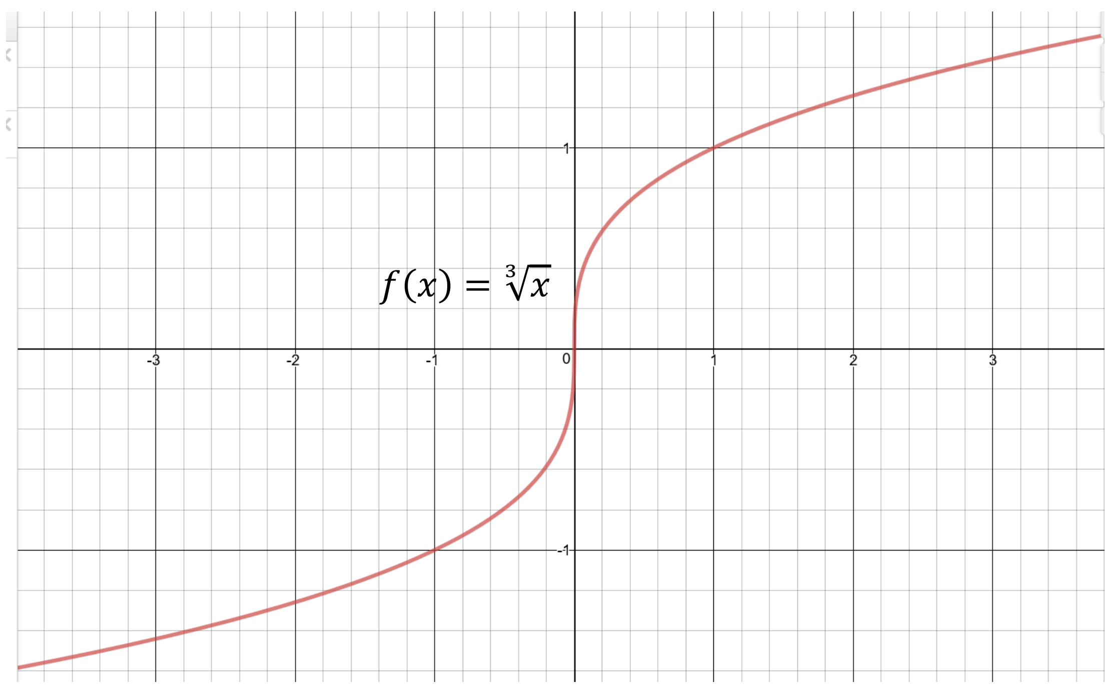
   Ex. $f(x) = |x|$, at $x = 0$, or $f(x) = \sqrt{|x|}$, at $x = 0$.





3. f has a vertical tangent at $x = a$.

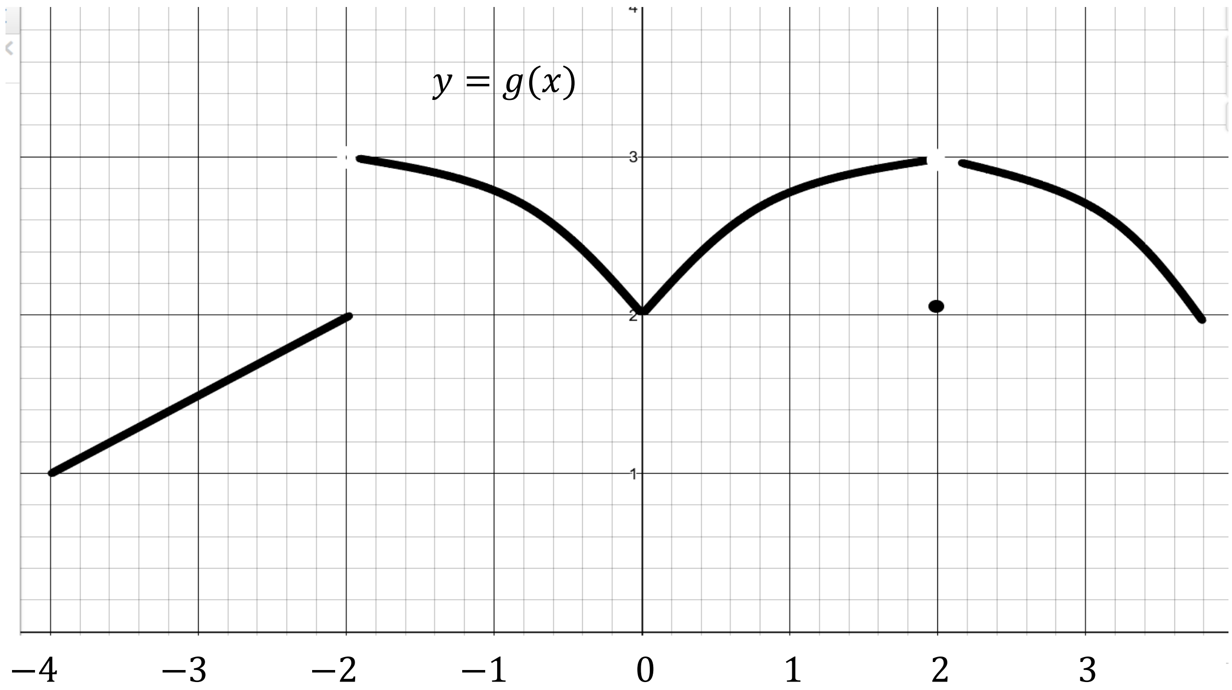
   Ex. $f(x) = \sqrt[3]{x}$ at $x = 0$.



Ex. a. Find the values of x in $(-4, 3)$ at which $g(x)$ is not continuous.

b. Find the values of x in $(-4, 3)$ at which $g(x)$ is not differentiable.

c. Sketch a rough graph of $g'(x)$.



a. $g(x)$ is not continuous at $x = -2,2$.

b. $g(x)$ is not differentiable at $x = -2,0,2$.

c. Rough graph of $g'(x)$.