

Partial Derivatives and Derivatives-HW Problems

1. Find all of the partial derivatives of the following functions.

a. $f(x, y) = \cos(xy)$

b. $g(x, y) = \cos[xsiny]$

c. $f(x, y, z) = y^z$

d. $h(x, y, z) = z^{(x-y)}$

2. Let $g: \mathbb{R}^2 \rightarrow \mathbb{R}$ by

$$g(x, y) = \begin{cases} xy \left(\frac{x^2 - y^2}{x^2 + y^2} \right) & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

a. Show that $\frac{\partial g}{\partial y}(x, 0) = x$ for all x and $\frac{\partial g}{\partial x}(0, y) = -y$ for all y .

b. Using the limit definition of a partial derivative show that

$$\frac{\partial^2 g}{\partial x \partial y}(0, 0) \neq \frac{\partial^2 g}{\partial y \partial x}(0, 0).$$

3. Let $f(x, y) = \begin{cases} x + \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

- i. Determine where the partial derivatives, $f_x(x, y)$ and $f_y(x, y)$, exist and find their values.
- ii. Determine if $f(x, y)$ is continuous at $(0, 0)$.
- iii. Determine if $f(x, y)$ is differentiable at $(0, 0)$.

4. Let $f(x, y) = (xy + xe^y, x\cos(y), e^{xy})$ and $h(x, y) = e^{(x+y)}$.
Find $Df(x, y)$, $Dh(x, y)$, and $D(h(x, y))(f(x, y))$ at $(x, y) = (1, 0)$.
5. Let $f(x, y) = \frac{xy^3}{x^3+y^6}$ if $(x, y) \neq (0, 0)$
 $= 0$ if $(x, y) = (0, 0)$.
- Find $f_x(x, y)$, $f_y(x, y)$ for all $(x, y) \in \mathbb{R}^2$.
 - Determine if $f(x, y)$ is continuous at $(0, 0)$.
 - Determine if f is differentiable at $(0, 0)$.
6. Let $f(x, y) = (2x + 3y, xy^2)$ and
 $g(u, v, w) = (e^u + v + w, e^{(v-w)} + w)$.
- Find $Df(x, y)$ and $Dg(u, v, w)$.
 - Let $h(u, v, w) = f(g(u, v, w))$. Use the chain rule to find $Dh(0, -2, -2)$.