The Metric Space $C(I)$- HW Problems

1. Consider the metric space $C[0,1]$ with the metric $d(f,g) = \sup_{0\le x\le 1} |f(x) - g(x)|$. Let

$$E = N_{\frac{1}{2}}(x^2) = \{f \in C[0,1] \mid d(x^2, f) < \tfrac{1}{2}\}.$$

(This is the same as the ball of radius $\frac{1}{2}$ centered at x^2, $B_{\frac{1}{2}}(x^2)$)

a. Is $g(x) = x^3 \in E$? Prove your answer.

b. Give an example of a subset of $C[0,1]$ that is totally bounded (don't use parts c or d of this problem).

c. Let $H = \{f_n(x) = \frac{x}{n}\ ;\ n \in \mathbb{Z}^+,\ 0 \le x \le 1\} \subseteq C[0,1]$. Is H totally bounded? Give justification for your answer.

d. Prove $J = \{f(x) = c \mid 0 \le c \le 2,\ \ 0 \le x \le 1\} \subseteq C[0,1]$ is totally bounded. Hint: consider problem #4 in the HW on totally bounded sets.