Fourier Series: The L_2 Norm and Calculating Fourier Series-
HW Problems

1. Let $f: \mathbb{R} \to \mathbb{R}$ be 2π periodic and $f \in R[-\pi, \pi]$.

a. Show that if f is even then its Fourier series can be written with only cosine terms (and possibly a constant term).

b. Show that if f is odd then its Fourier series can be written with only sine terms.

In problems 2-4 find the Fourier series for $f(x)$ (Extend each function to $\mathbb{R}$ by making it periodic on its domain).

2. $f(x) = 0$ if $-\pi \leq x \leq 0$

$= 4$ if $0 < x < \pi$

3. $f(x) = x$ if $-\pi \leq x < \pi$

4. $f(x) = |\sin(x)|$ if $-\pi \leq x \leq \pi$

(Use the fact that

$$(\sin(x))(\cos(nx)) = \frac{1}{2}[\sin[(n+1)x] - \sin[(n-1)x]]).$$

5a. Find the Fourier series for $f(x) = (\pi - x)^2; \quad 0 \leq x \leq 2\pi.$

b. Show that this Fourier series converges uniformly on $\mathbb{R}$.

c. Evaluate $f(x)$ and the Fourier series at $x = 0$ to show $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$