

The Riemann Integral- HW Problems

1. Evaluate $\int_0^2 x^3 d\alpha$ if $\alpha(x) = x^2$ on $[0,2]$.
2. Suppose $f \in C[a, b]$ and $\int_a^x f(t)dt = 0$ for all $x \in [a, b]$. Show that $f(x) = 0$ for all $x \in [a, b]$.
3. Suppose $f, g \in C[a, b]$ and $\int_a^x f(t)dt = \int_a^x g(t)dt$ for all $x \in [a, b]$. Prove that $f(x) = g(x)$ for all $x \in [a, b]$.
4. Let $F(x) = \int_0^x f(t)dt$. Determine for what values of $x \in \mathbb{R}$ that $F'(x)$ exists if
 - a. $f(x) = 0$ if $x < 0$
 $= 1$ if $x \geq 0$
 - b. $f(x) = 0$ if $x < 0$
 $= x$ if $x \geq 0$
 - c. $f(x) = 1$ if $x = 1$
 $= 0$ if $x \neq 1$