

The Space $R_\alpha[a, b]$ - HW Problems

1. Suppose that $f \in C[a, b]$ and $f(x) \geq 0$ for all $x \in [a, b]$. Show that if $f(p) > 0$ for some $a < p < b$ then $\int_a^b f(x)dx > 0$ (Riemann integral). Thus we can conclude that $\|f\| = \int_a^b |f(x)|dx$ defines a norm on $C[a, b]$ since $|f(x)| \geq 0$ and $\|f\| = 0$ if and only if $f(x) = 0$.

2. Prove $|\int_0^{2\pi} \frac{\cos(x)}{\sqrt{x^2+1}} dx| \leq \sqrt{\pi \tan^{-1}(2\pi)}$

3. Prove $\int_0^1 2(\sqrt{x})(\sqrt{1+8x^3})dx \leq \sqrt{6}$.