Limits of Functions- HW Problems

1. Prove the following limits from the δ, ϵ definition of $\lim_{x \to p} f(x) = q$.

a. $\lim_{x \to 2} (3x - 4) = 2$

b. $\lim_{x \to 4} (x^2 - 2x) = 8$

c. $\lim_{x \to 5} \left(\frac{x^2 - 25}{x - 5} \right) = 10$

d. $\lim_{x \to 2} (x^3) = 8$

e. $\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2$

f. $\lim_{x \to 0} \left(\sqrt[3]{x} \right) = 0$

g. $\lim_{(x,y) \to (0,0)} (x^2 - 2y^2 + 1) = 1$

2. Prove (hint: use the squeeze theorem):

a. $\lim_{x \to 0} \left(x \cos\left(\frac{1}{x} \right) \right) = 0$

b. $\lim_{x \to 0} \left(x \tan^{-1}(x) \right) = 0$

3. Assume $\lim_{x \to a} f(x) = L$. If $f(x) \geq k$, prove that $L \geq k$. (Hint: assume that $L < k$ and use a δ, ϵ argument to get a contradiction).