Subsequences and Cauchy Sequences- HW Problems

1. Find a sequence of real numbers $\{a_i\}$ such that $\lim_{n\to\infty} a_n$ does not exist, but it has 3 subsequential limits of $0,1,2$.

2. Assume $\{p_n\}$, and $\{r_n\}$ are Cauchy sequences in $\mathbb{R}$. Using the definition of a Cauchy sequence prove that $\{p_n + r_n\}$ is a Cauchy sequence in $\mathbb{R}$.

3. Using the definition of a Cauchy Sequence, prove that $\{\frac{1}{n^2}\}$ is a Cauchy sequence in $\mathbb{R}$.

4. Suppose that $\{a_i\}$ is a Cauchy sequence in a metric space X, d and $\lim_{n\to\infty} a_n = p$. Suppose, in addition, $\{b_i\}$ is a sequence such that $d(a_n, b_n) < \frac{1}{n}$ for all positive integers n. Prove that $\lim_{n\to\infty} b_n = p$.

5. Give an example of a metric space X, d and a sequence $\{p_n\} \subseteq X$ such that $\{p_n\}$ is a Cauchy sequence in X, d but $\{p_n\}$ doesn't converge in X, d (ie X, d is not a complete metric space). Use ϵ-N arguments to show that your sequence is Cauchy and that the limit is not in X, d.