Sequences- HW Problems

1. Using the ϵ-N definition of a convergent sequence, prove the following (you cannot use any theorems about convergent sequences):

a. $\lim\limits_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2}$

b. $\lim\limits_{n\to\infty} \frac{1}{n^2} = 0$

c. $\lim\limits_{n\to\infty} \ln\left(\frac{n+1}{n}\right) = 0$

d. $\lim\limits_{n\to\infty} \ln\left(\frac{n}{n+1}\right) = 0$

e. $\lim\limits_{n\to\infty} e^{\left(\frac{n+1}{n}\right)} = e$ ($\frac{n+1}{n}$ is the exponent of e)

2. Using the definition of a convergent sequence, prove $\lim\limits_{n\to\infty} \frac{2n}{3n+1} \neq \frac{1}{3}$.

3. Let $\{a_i\}$ be a sequence in $\mathbb{R}$. Prove $\lim\limits_{n\to\infty} a_n = 0$, if and only if, $\lim\limits_{n\to\infty} |a_n| = 0$.

If $\lim\limits_{n\to\infty} |a_n| = 1$, is it true that $\lim\limits_{n\to\infty} a_n = 1$? Prove your answer.

4. Let $\{a_i\}$ be a sequence in a metric space X,d. Prove with an ϵ-N argument that $\lim\limits_{n\to\infty} a_n = p$ if and only if $\lim\limits_{n\to\infty} d(a_n, p) = 0$.

5. $\{a_i\}$ is a sequence in $\mathbb{R}$ such that $\lim_{n\to\infty} a_n = 0$. Suppose that $\{b_i\}$ is a sequence in $\mathbb{R}$ such that $|b_i| \leq M$, for all i where $M \geq 0$. Prove from the definition of a limit of a sequence that $\lim_{n\to\infty} (a_n b_n) = 0$.

6. Suppose $\{p_i\}$ and $\{q_i\}$ are sequences of real numbers such that $\lim_{n\to\infty} p_n = p$ and $\lim_{n\to\infty} q_n = q$. Give an ϵ-N proof that $\lim_{n\to\infty} (p_n + 4q_n) = p + 4q$.