Open and Closed Sets in a Metric Space- HW Problems

1. Make a table of the subsets of $\mathbb{R}^2$ below (using the standard metric on $\mathbb{R}^2$) where across the top you have the categories: "Limit Points", "Isolated Points", "Bounded", "Closed", and "Open", and along the left side you have the sets A through I. Identify all limit points and isolated points. Put "Y" or "N" for the rest.

$A = \{(x, y) |\, x^2 + y^2 \leq 1\}$

$B = \{(x, y) |\, 0 < x^2 + y^2 \leq 1\}$

$C = \{(x, y) |\, 0 < x^2 + y^2 < 1\}$

$D = \{(x, y) |\, \frac{1}{2} < x^2 + y^2 < 1\} \cup \{(0,0)\}$

$E = \{(x, y) |\; 0 < x < 2, \quad y = 1\}$

$F = \{(x, y) |\; 0 \leq x \leq 2, \quad 0 \leq y \leq 1\}$

$G = \{(x, y) |\; 1 \leq x, \quad 1 \leq y \leq 2\}$

$H = \{(0,0), (0,1), (1,0)\}$

$I = \{(x, y) |\; y = \sin(x)\}$

2. Prove the following (assume the standard metric on $\mathbb{R}$ and $\mathbb{R}^2$):

   a. $(-2,2)$ is an open set in $\mathbb{R}$.

   b. $[-2,2]$ is a closed set in $\mathbb{R}$.

   c. $(-2,2]$ is neither an open set nor a closed set in $\mathbb{R}$.

   d. Is $A = \{(x, y) |\, -2 < x < 2, \;\; y = 0\}$ open in $\mathbb{R}^2$? Prove your answer.

3. Let $A, B, C \subseteq X, d$ be non-empty open sets in a metric space X. Prove the following (without using the theorem that states that the union of open sets is open and the finite intersection of open sets is open).

a. $A \cup B \cup C$ is open in X.

b. $A \cap B \cap C$ is open in X.

4. Prove that If X, d is a metric space and $E \subseteq F \subseteq X$, then $\bar{E} \subseteq \bar{F}$.