

Metric Spaces: Definitions and Examples- HW Problems

1. Prove from the definition of a metric space that \mathbb{R}^2, d is a metric space where

$$d(p, q) = |p_1 - q_1| + |p_2 - q_2|; \quad p = (p_1, p_2), \quad q = (q_1, q_2).$$

2. Let $A = \{\text{positive Integers}\}$ and $B = \{\text{all Integers}\}$. Let d be defined by: $d(p, q) = |p^2 - q^2|$.

- a. Prove that A, d is a metric space.
- b. Prove that B, d is not a metric space.

3. Prove that $d((x_1, y_1), (x_2, y_2)) = |x_2 - x_1|$ is not a metric on \mathbb{R}^2 .

4a. Define a metric on \mathbb{R} by $d(p, q) = |e^p - e^q|$. Find all of the points $p \in \mathbb{R}$ such that $d(p, 2) < 1$.

b. Using the metric in problem number 1, find the set of all points $p = (p_1, p_2) \in \mathbb{R}^2$ such that $d(p, O) \leq 1$, where $O = (0, 0)$. Sketch this set in \mathbb{R}^2 .

5. Let $X = C[0,1]$ be the set of real valued, continuous functions on $[0,1]$. Define 2 metrics on X by

$$d_1(f(x), g(x)) = \int_0^1 |f(x) - g(x)| dx \text{ and}$$

$$d_2(f(x), g(x)) = \max_{x \in [0,1]} |f(x) - g(x)|.$$

Let $f(x) = x$ and $g(x) = x^2$.

Find $d_1(f(x), g(x))$, and $d_2(f(x), g(x))$.