Finite, Countable, and Uncountable Sets- HW Problems

1. Prove the following sets are Countable (i.e., there is a 1-1 mapping onto the set $J = \{1,2,3,4, \dots\}$)

   a. $A = \{-2, -4, -6, -8, \dots\}$

   b. $B = \{-1, -3, -5, -7, \dots\}$

   c. $C = \{-1, -4, -9, -16, \dots\}$

2. Show that sets B and C are equivalent to $A = \{x \epsilon \mathbb{R} |\ 0 < x < 1\}$

   a. $B = \{x \epsilon \mathbb{R} |\ 0 < x < 10\}$

   b. $C = \{x \epsilon \mathbb{R} |\ -4 < x < -1\}$

3a. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$. Find $f^{-1}(16)$ and $f^{-1}(U)$, where $U = [9,16]$.

b. $f: \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x^2 + y^2$. Find $f^{-1}(0)$, $f^{-1}(1)$, and $f^{-1}(U)$, where $U = (1,4)$.

4. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ and let $U = (-1, 1)$.

   a. Find $f^{-1}(U)$.

   b. Find $f(f^{-1}(U))$. (Notice that $f(f^{-1}(U)) \subseteq U$, but $f(f^{-1}(U)) \neq U$)