Group Homomorphisms- HW Problems

For problems 1-6 determine whether the maps are group homomorphisms.

1. $\phi: \mathbb{R} \to \mathbb{R}$ under addition, $\phi(x) = 2x + 1$.
2. $\phi: \mathbb{R} \to \mathbb{R}^*$; $\mathbb{R}$ under addition, $\mathbb{R}^*$ under multiplication, $\phi(x) = 3^x$
3. G any group. $\phi: G \to G$ by $\phi(g) = g^{-1}$, $g \in G$.
4. $\phi: M_n(\mathbb{R}) \to \mathbb{R}$ by $\phi(A) = \det(A)$
5. Let $\mathfrak{J}$ be the group of real-valued infinitely differentiable functions on $\mathbb{R}$ (under addition). $\phi: \mathfrak{J} \to \mathfrak{J}$ by $\phi(f) = f'(x)$; $f \in \mathfrak{J}$.
6. Let $\mathfrak{J}$ be the group of real-valued infinitely differentiable functions on $\mathbb{R}$. $\phi: \mathfrak{J} \to \mathfrak{J}$ by $\phi(f) = f''(x) + 2f'(x) + f(x)$; $f \in \mathfrak{J}$.

For problems 7 and 8 find the following quantities for the homomorphism ϕ.

7. $\ker(\phi)$ and $\phi(12)$ for $\phi: \mathbb{Z} \to \mathbb{Z}_{14}$ with $\phi(1) = 6$.
8. $\ker(\phi)$ and $\phi(-4,3)$ for $\phi: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ with $\phi(1,0) = 2$ and $\phi(0,1) = -3$.

9. Let G be a group and $g \in G$. For which $g \in G$ is $\phi: G \to G$ by $\phi(x) = xg$ a homomorphism?

10. Let $\phi: \mathbb{Z}_7^* \to \mathbb{Z}_7^*$ where $\mathbb{Z}_7^*$ is the multiplicative group of elements in $\mathbb{Z}_7$ and $\phi(x) = x^3 \ (mod\ 7)$. Find the kernel of ϕ.

11. Let $\phi: G_1 \to G_2$ be a group homomorphism. Prove $\phi[G_1]$ is abelian if and only if for all $x, y \in G_1$, $xyx^{-1}y^{-1} \in \ker(\phi)$.

12. Suppose $\phi: G_1 \to G_2$ and $\tau: G_2 \to G_3$ are group homomorphisms. Prove that $\tau \circ \phi: G_1 \to G_3$ is a group homomorphism.

13. Let G be a group and g any element in G. Prove $\phi: \mathbb{Z} \to G$ by $\phi(n) = g^n$ is a group homomorphism.