

Finitely Generated Abelian Groups- HW Problems

1. List all of the elements of the group $\mathbb{Z}_2 \times \mathbb{Z}_3$ and find the order of each element.

For problems 2-4 find the order of the given element in each group.

2. $(2, 8)$ in $\mathbb{Z}_4 \times \mathbb{Z}_{18}$
3. $(2, 12, 10)$ in $\mathbb{Z}_8 \times \mathbb{Z}_{24} \times \mathbb{Z}_{16}$
4. $(2, 8, 10)$ in $\mathbb{Z}_8 \times \mathbb{Z}_{10} \times \mathbb{Z}_{24}$

For problems 5-7 find the order of the largest cyclic subgroup of the given group.

5. $\mathbb{Z}_{12} \times \mathbb{Z}_{18}$
6. $\mathbb{Z}_8 \times \mathbb{Z}_{20} \times \mathbb{Z}_{16}$
7. $\mathbb{Z}_8 \times \mathbb{Z}_{10} \times \mathbb{Z}_{24}$

For problems 8-10 determine if the groups given are isomorphic. Explain your answer.

8. $\mathbb{Z}_4 \times \mathbb{Z}_{16}$ and $\mathbb{Z}_8 \times \mathbb{Z}_8$
9. $\mathbb{Z}_{10} \times \mathbb{Z}_{70}$ and $\mathbb{Z}_{10} \times \mathbb{Z}_{10} \times \mathbb{Z}_7$
10. $\mathbb{Z}_{10} \times \mathbb{Z}_{70}$ and $\mathbb{Z}_{20} \times \mathbb{Z}_{35}$

For problems 11-13 find all abelian groups up to isomorphism of the given order.

11. 36

12. 360

13. 720

Determine whether the following statements are true or false.

14. $\mathbb{Z}_2 \times \mathbb{Z}_6$ is isomorphic to \mathbb{Z}_{12} .

15. $\mathbb{Z}_2 \times \mathbb{Z}_7$ is isomorphic to \mathbb{Z}_{14} .

16. $\mathbb{Z}_2 \times \mathbb{Z}_3$ is isomorphic to S_3 .

17. $|\mathbb{Z}_6 \times \mathbb{Z}_9| = 18$.

18. If $|G| = 11$ then G is abelian.

19. A_3 is an abelian group.