Cosets and Lagrange's Theorem- HW Problems

Find all cosets of the subgroup H of the group G.

1. $H = 6\mathbb{Z}, \quad G = 2\mathbb{Z}.$
2. $H =< 3 > , \quad G = \mathbb{Z}_{18}$

For problems 3-6 find the index of the subgroup H in the group G.

3. $H = 6\mathbb{Z}, \quad G = 2\mathbb{Z}.$
4. $H =< 3 > , \quad G = \mathbb{Z}_{18}$
5. $H =< \sigma >$, where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 5 & 4 & 3 \end{pmatrix}$, $G = S_6$.
6. $H =< \sigma >$, where $\sigma = (1,\ 5,\ 4,\ 2)(2,\ 3) \in S_5, \quad G = S_5$.

7. The dihedral group D_4 has 8 elements.

$\rho_0 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$ $\qquad \mu_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

$\rho_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ $\qquad \mu_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$

$\rho_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ $\qquad \delta_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$

$\rho_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ $\qquad \delta_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 \end{pmatrix}$

$H = \{\rho_0, \mu_1\}$ is a subgroup of D_4.

a. Find all disjoint left cosets of H.

b. Find all disjoint right cosets of H.

8. Let H be a subgroup of a group G. Suppose that $g^{-1}hg \in H$ for all $g \in G$ and $h \in H$. Show that $gH = Hg$ for all $g \in G$.

9. Suppose $|G| = pq$ where p and q are prime numbers. Show every proper subgroup of G is a cyclic group.

10. Show that if $|G| = n$ (ie G has finite order) and e is the identity element of G then $a^n = e$ for all $a \in G$.

11. Suppose H is a subgroup of a group G of index 2. Prove every left coset is a right coset.