Permutation Groups- HW Problems

For problems 1-4.

Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 5 & 4 & 3 \end{pmatrix}$; $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 1 & 5 & 3 & 2 \end{pmatrix}$.

Find:

1. $\tau \sigma$

2. $\tau^2 \sigma$

3. $\sigma^{-1} \tau$

4. Find the orbit of 1 under σ given by $O_{\sigma,1} = \{\sigma^n(1) | n \in \mathbb{Z}\}$

5. Find the number of elements in $\{\sigma \in S_4 | \sigma(2) = 3\}$.

6. Find all subgroups of S_3 and draw the subgroup diagram.

Determine which of the following functions are permutations of $\mathbb{R}$.

7. $f(x) = 2x + 1$

8. $f(x) = x^4$

9. $f(x) = e^{-x}$.

10. Let $a, b \in A$ and $\sigma \in S_A$. Let $O_{\sigma,a} = \{\sigma^n(a) | n \in \mathbb{Z}\}$ and $O_{\sigma,b} = \{\sigma^n(b) | n \in \mathbb{Z}\}$. Show that if $O_{\sigma,a} \cap O_{\sigma,b} \neq \phi$ then $O_{\sigma,a} = O_{\sigma,b}$.

11. Show that S_n is nonabelian for $n \geq 3$.