# Maximal and Prime Ideals- HW Problems

In problems 1-3 find all prime ideals and maximal ideals in the given ring.

1. $\mathbb{Z}_6$
2. $\mathbb{Z}_{18}$
3. $\mathbb{Z}_2 \times \mathbb{Z}_8$

In problems 4-6 find all $a$ in the given ring such that the factor ring is a field.

4. $\mathbb{Z}_3[x]/ < (x^3 + 2x^2 + a) >; \ \ a \in \mathbb{Z}_3$
5. $\mathbb{Z}_3[x]/< (x^3 + ax + 1) >; \quad a \in \mathbb{Z}_3$
6. $\mathbb{Z}_5[x]/< (x^2 + 2x + a) >; \quad a \in \mathbb{Z}_5$.

In problems 7-13 determine if the statement is true or false.

7. Let $R$ be a commutative ring with unity and $N \neq R$ an ideal in $R$. Then $R/N$ is an integral domain if and only if $N$ is a maximal ideal.
8. Let $R$ be a commutative ring with unity. Then $M$ is a maximal ideal in $R$ if and only if $R/M$ is a field.
9. The only ideals in $\mathbb{Q}$ are $\mathbb{Q}$ and $\{0\}$.
10. $\mathbb{Q}[x]/< (x^2 - 4) >$ is a field.
11. Every ideal of $\mathbb{Z}$ is a principal ideal.

12. Every maximal ideal of a commutative ring with unity is a prime ideal.

13. If $F$ is a field then every ideal in $F[x]$ is a principal ideal.

14. Find a prime ideal in $\mathbb{Z} \times \mathbb{Z}$ that is not maximal.

15. Determine if $\mathbb{Q}[x]/< (x^2 - 4x + 3) >$ is a field. Explain your answer.