Fermat's Little Theorem and Euler's Theorem- HW Problems

1. Find a generator for the multiplicative group of non-zero elements of \mathbb{Z}_{13} .

For problems 2-5 evaluate the expression.

- 2. 3⁹⁰ (mod 23)
- 3. $2^{75} \pmod{19}$
- 4. $(7^{602} + 1) \pmod{18}$
- 5. $(7^{62} + 3) \pmod{18}$

For problems 6-8 find all solutions in \mathbb{Z} to the equations.

- 6. $27x \equiv 5 \pmod{8}$
- 7. $26x \equiv 5 \pmod{8}$
- 8. $33x \equiv 12 \pmod{24}$

9. Let p be a prime number. Show that in the field \mathbb{Z}_p , 1 and p-1 are the only elements that are their own inverses, ie the only solutions of $x^2 - 1 \equiv 0 \pmod{p}$ are 1 and p - 1.