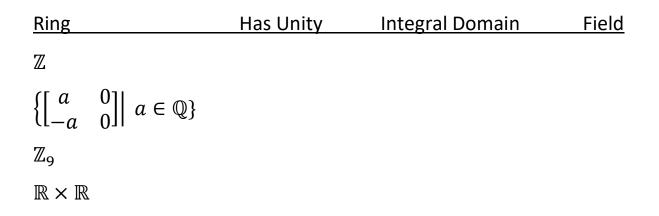
Integral Domains- HW Problems

For problems 1-4 solve the equations in the given ring/field.

- 1. 4x = 3 in \mathbb{Z}_7
- 2. 4x = 3 in \mathbb{Z}_{19}
- 3. $x^2 + 4x + 1 = 0$ in \mathbb{Z}_6
- 4. $x^2 + 4x + 2 = 0$ in \mathbb{Z}_6
- 5. Show that $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ is a zero divisor in $M_2(\mathbb{R})$.

6. Put a "Y" if the ring has the property and an "N" if it doesn't. Assume the usual addition and multiplication.



7. Find all zero divisors in \mathbb{Z}_{12} .

For problems 8-12 give an example of a ring with the given property.

- 8. An integral domain that is not a field.
- 9. A commutative ring with unity that's not an integral domain.
- 10. A commutative ring without unity.
- 11. A non-commutative ring with unity.
- 12. A non-commutative ring without unity.

13. Show that in a division ring there are exactly two elements x such that $x^2 = x$ (called idempotent elements).

14. Let D_1 and D_2 be subdomains of an integral domain D. Show that

$$D_1 \cap D_2 = \{ d \in D \mid d \in D_1, d \in D_2 \}$$

is a subdomain of *D*.