Integral Domains- HW Problems

For problems 1-4 solve the equations in the given ring/field.

1. $4x = 3$ in $\mathbb{Z}_7$
2. $4x = 3$ in $\mathbb{Z}_{19}$
3. $x^2 + 4x + 1 = 0$ in $\mathbb{Z}_6$
4. $x^2 + 4x + 2 = 0$ in $\mathbb{Z}_6$

5. Show that $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ is a zero divisor in $M_2(\mathbb{R})$.

6. Put a "Y" if the ring has the property and an "N" if it doesn't. Assume the usual addition and multiplication.

Ring	Has Unity	Integral Domain	Field
$\mathbb{Z}$			
$\left\{\begin{bmatrix} a & 0 \\ -a & 0 \end{bmatrix} \middle\vert\, a \in \mathbb{Q}\right\}$			
$\mathbb{Z}_9$			
$\mathbb{R} \times \mathbb{R}$			

7. Find all zero divisors in $\mathbb{Z}_{12}$.

For problems 8-12 give an example of a ring with the given property.

8. An integral domain that is not a field.

9. A commutative ring with unity that's not an integral domain.

10. A commutative ring without unity.

11. A non-commutative ring with unity.

12. A non-commutative ring without unity.

13. Show that in a division ring there are exactly two elements x such that $x^2 = x$ (called idempotent elements).

14. Let D_1 and D_2 be subdomains of an integral domain D. Show that

$$D_1 \cap D_2 = \{d \in D \mid d \in D_1,\ d \in D_2\}$$

is a subdomain of D.