Rings and Fields- HW Problems

For problems 1 and 2 find the product in the given ring.

1. $(18)(2)$ in $\mathbb{Z}_{24}$.
2. $(3,-4)(-5,3)$ in $\mathbb{Z}_{10} \times \mathbb{Z}_8$

For problems 3-5 identify which of the sets are rings.

3. $\mathbb{R}^+$ with the usual addition and multiplication.
4. $4\mathbb{Z} \times \mathbb{Z}$ with addition and multiplication componentwise.
5. $\{a + b\sqrt{2} \mid a, b \in \mathbb{Q}^*\}$ with the usual addition and multiplication.

For problems 6-8 identify all of the units in the given ring.

6. $\mathbb{Q} \times \mathbb{Z}$
7. $\mathbb{Q} \times \mathbb{Z} \times \mathbb{Z}_4$
8. $\mathbb{Z}_4 \times \mathbb{R} \times \mathbb{Z}$

9. Let $\phi: M_n(\mathbb{R}) \to \mathbb{R}$, $n > 1$; by $\phi(A) = \det(A)$. Either prove that ϕ is a ring homomorphism or show why it isn't.

10. Show that the rings $2\mathbb{Z}$ and $5\mathbb{Z}$ are not isomorphic.

11. Show that $x^2 - y^2 = (x-y)(x+y)$ for all x, y in a ring R if and only if R is commutative.

12. Let p be a prime number. Use the binomial theorem to show that in $\mathbb{Z}_p$: $(x+y)^p = x^p + y^p$ for all $x, y \in \mathbb{Z}_p$.

13. Let $R_1$and R_2 be subrings of a ring R. Prove that

$$R_1 \cap R_2 = \{x \in R \mid x \in R_1,\ x \in R_2\}$$

is a subring of R.